NATURAL FREQUENCIES OF STEEL BEAMS

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NATURAL FREQUENCIES
OF STEEL BEAMS

HERMAN M. BADING

NATURAL FREQUENCIES OF STEEL BEAMS

by

Herman Morris Bading
//
Lieutenant, United States Navy

Submitted in partial fulfillment
of the requirements for the

CERTIFICATE OF COURSE COMPLETION

IN

MECHANICAL ENGINEERING

United States Naval Postgraduate School
Monterey, Cal.

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IN

MECHANICAL ENGINEERING

FROM THE

UNITED STATES NAVAL POSTGRADUATE SCHOOL

PREFACE

A difference in frequency exists between a beam mounted under ideal conditions and a beam mounted under actual conditions.

It was the desire of the author to study the variation in frequencies between the actual and ideal beams.

An electromagnetic exciting device was used for the investigation to excite the beams.

The work was done by the author during the period January 1955 through April 1955, at the United States Naval Postgraduate School, Monterey, California.

The author is indebted to Professor E. K. Gatcombe for much helpful assistance in making the study.

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TABLE OF SYMBOLS

(Listed in the order of their use in the text)

ω_n	- Natural Frequency in Radians/second
f	- Natural Frequency in Cycles/second
l	- Length of Beam in Inches
E	- Young's Modulus of Elasticity psi
I	- Moment of Inertia (in^4)
μ	- Mass per Unit Length ($\frac{lb\ sec^2}{in^2}$)

CHAPTER 1

INTRODUCTION

A beam mounted and held under non-ideal conditions is known to have a natural frequency different from the natural frequency of the same beam mounted and held under ideal conditions.

It was the desire of the author to study the amount of this difference and to investigate the discrepancy between the observed experimental and ideal natural frequencies for a beam clamped at both ends.

A codicillary to this investigation was the desire to find a method other than mechanical to excite the beams, preferably to frequencies above their natural frequencies.

CHAPTER II

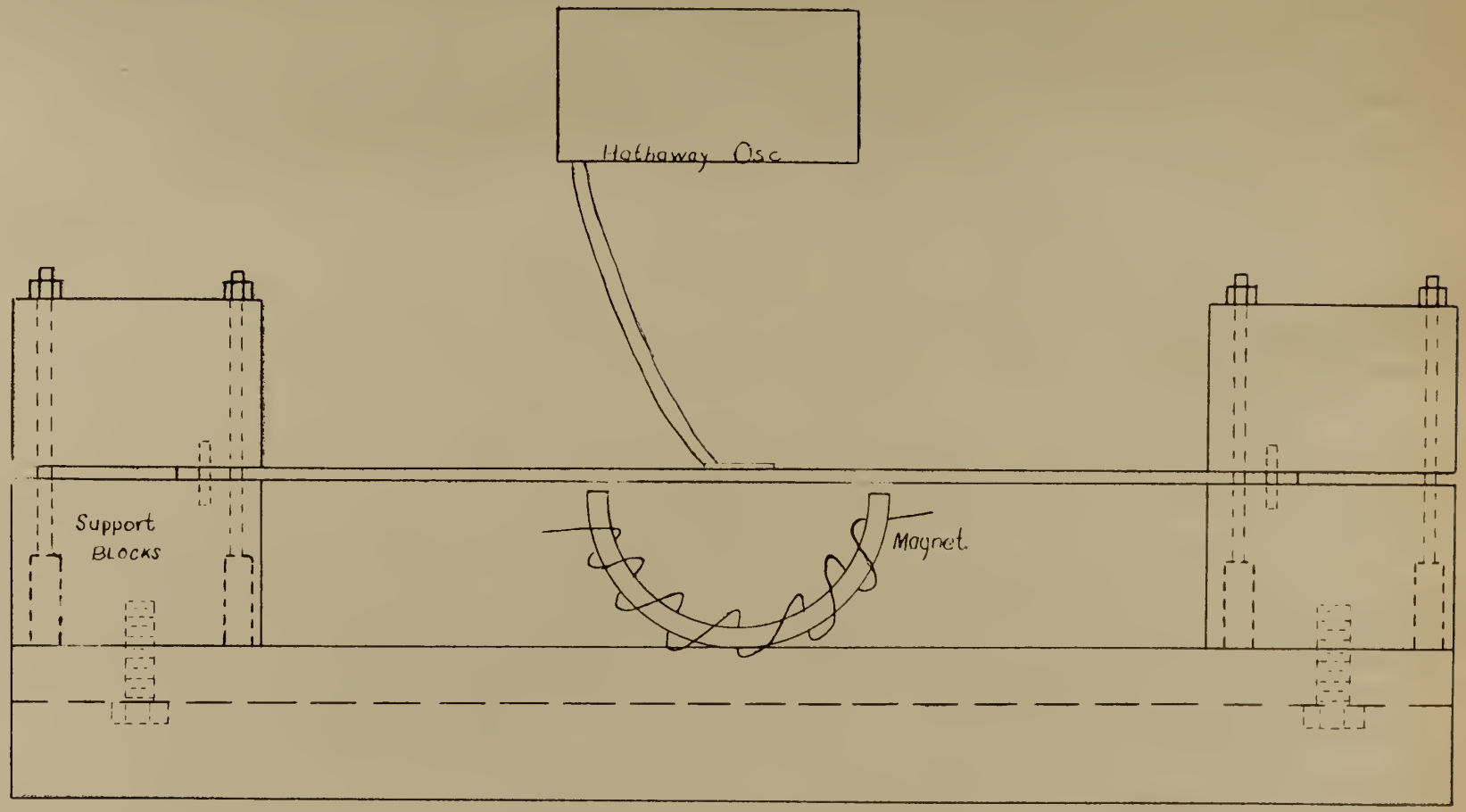
EXPERIMENTAL APPARATUS AND PROCEDURE

The experimental set up was as shown in Figure 1. Beams selected for testing were three cold rolled, mild steel beams one-eighth inches in thickness and one-half, three-quarters, and one inch respectively in width. Variation of length was accomplished by starting at the longest length and drilling holes successively inward toward the center of the beam, and moving the cast iron blocks with the securing dowels toward each other, fitting the dowels into corresponding holes as they were drilled.

End conditions were as follows: At each end two cast iron blocks four inches on a side and three inches thick contained the beam. A one-quarter inch dowel fit through the beam, into each block. Four one-quarter inch bolts passing through both blocks were taken up to clamp the beam securely. Each of the two bottom blocks was in turn bolted by a five-eighth inch bolt to a heavy cast iron table.

A magnet was built by the author to operate on 115^{V}, 10 amp. supply from a 400 cps motor-generator set. This magnet was mounted on the cast iron table under the beam which it excited.

An A-1 strain gage was mounted at the center on top of the beam and its output lead to the Hathaway Oscillograph.



Shematic of Setup (unscaled)

FIG. 1

The frequency of each beam was obtained by exciting the beam to 400 cps with the magnet, then de-energizing the magnet to allow the beam to vibrate at its natural frequency and recording this natural frequency on the Hathaway Oscillograph from the strain gage output.

It was hoped that the contribution of end conditions might be evaluated, but time prohibited the redesigning of the experimental setup and moving of the equipment to one of the large hydraulic testing machines. A substitute was devised however. Beams 3-1, 3-4 and 4 were setup in a large testing machine and varying force as shown in Tables 2, 3 and 4 applied to a ten inch I beam spanning the support blocks, causing them to constrain more firmly the beams. The beams were plucked to set up the vibrations. Sample traces are shown in Figures 3, 4 and 5. Results are considered unreliable because of their large deviation which is unexplained. Whether the deviation is caused by mounting or by the method of excitation was not determined.

CHAPTER III

COMPUTATIONS

Actual frequencies were taken directly from the Hathaway Oscillograph, trace samples of which are shown in Figure 2. Each small division shown represents .01 second regardless of photographic paper speed.

For computing the theoretical natural frequency, it was assumed that the beam vibrated in the fundamental mode. The following formula for a theoretical clamped-clamped beam was taken from "Mechanical Vibrations" by Den Hartog.

$$\omega_N = \frac{22.4}{l^2}\left(\frac{EI}{\mu}\right)^{\frac{1}{2}}$$

Analysis of the decay in amplitude of the wave form showed that the damping produced negligible effect on the frequency, hence it was not considered.

COMPUTATIONS

RUN 1-1

$$\omega_{N\ \text{Theor.}} = \frac{22.4}{(11.31)^2}\sqrt{\frac{(30 \times 10^6)\ (7.55 \times 10^{-5})\ (386)}{(.284)\ (5.89 \times 10^{-2})}} = 1270 \text{ rads/sec}$$

$$f_{\text{Theor.}} = \frac{1270}{2\pi} = 202 \text{ cps}$$

RUN 2-3

$$\omega_{N\ \text{Theor.}} = \frac{22.4}{(9.57)^2}\sqrt{\frac{(30 \times 10^6)\ (11.62 \times 10^{-5})\ (386)}{(.284)\ (8.96 \times 10^{-2})}} = 1775 \text{ rads/sec}$$

$$f_{\text{Theor.}} = \frac{1775}{2\pi} = 283 \text{ cps}$$

RUN 2-4

$$\omega_{N\ \text{Theor.}} = \frac{22.4}{(9.02)^2}\sqrt{\frac{(30 \times 10^6)\ (11.62 \times 10^{-5})\ (386)}{(.284)\ (8.96 \times 10^{-2})}} = 1997 \text{ rads/sec}$$

$$f_{\text{Theor.}} = \frac{1997}{2\pi} = 317 \text{ cps}$$

RUN 3-1

$$\omega_{N\ \text{Theor.}} = \frac{22.4}{(11.33)^2}\sqrt{\frac{(30 \times 10^6)\ (15.19 \times 10^{-5})\ (386)}{(.284)\ (11.9 \times 10^{-2})}} = 1263 \text{ rads/sec}$$

$$f_{\text{Theor.}} = \frac{1263}{2\pi} = 201 \text{ cps}$$

THEORETICAL NATURAL FREQUENCY COMPUTATIONS TAKING INTO ACCOUNT SHEAR AND ROTARY INERTIA

RUN 2-4 EMPLOYED FOR THE PURPOSE FROM KRUSZEWSKI'S WORK

$$K_s = \frac{1}{(\frac{8.02}{2})}\sqrt{\frac{(30 \times 10^6)\ (116.2 \times 10^{-6})}{(8.96 \times 10^{-2})\ (12 \times 10^6)}} = 1.266 \times 10^{-2}$$

$$K_s^2 = 1.6 \times 10^{-4}$$

$$K_{RI} = \frac{1}{(\frac{9.02}{2})}\sqrt{\frac{(116.2 \times 10^{-6})}{(8.96 \times 10^{-2})}} = .798 \times 10^{-2} \qquad K_{RI}^{\ 2} = 6.37 \times 10^{-5}$$

$$K_B = \omega\sqrt{\frac{(6.59 \times 10^{-5})\ (\frac{9.02}{2})^4}{(30 \times 10^6)\ (116.2 \times 10^{-6})}} = (2.8 \times 10^{-3})\,\omega$$

The equations for α and β as shown in Appendix III reduce simply to $\beta = \alpha = \sqrt{\frac{1}{K_B}}$ if it is noted that for the frequency with which I am concerned (i.e. $\approx$ 2000 rads/sec)

$$\frac{4}{K_B^{\ 2}} >> (K_s^{\ 2} - K_{RI}^{\ 2})^2 \text{ and } (K_s^{\ 2} \text{ plus } K_{RI}^{\ 2}).$$

The frequency equation shown as a 4th order determinant in Appendix III was solved by trial and error;

Assuming ω = 1997 rads/sec

$K_B = 5.59$ $\quad \alpha = \beta = .423$ $\quad 2K_B\alpha = 2K_B\beta = 4.74$

$\sin 2K_B\beta = -.999$ $\quad \sinh 2K_B\alpha = 57.21$

$\cos 2K_B\beta = .035$ $\quad \cosh 2K_B\alpha = 57.22$

$$\frac{K_B\alpha}{L} = \frac{K_B\beta}{L} = .525$$

$$\begin{vmatrix} 1 & 0 & 1 & 0 \\ 57.22 & 57.21 & .035 & -.999 \\ 0 & .525 & 0 & .525 \\ 30 & 30.05 & .5245 & .0184 \end{vmatrix}$$

$= -.275 + .552 - 15.75 + .553 + 901 + 15.73 - 902$

$= -.29$

FROM TIMOSHENKO'S WORK

Assume p = 1997 rads/sec

$$k = \left[\frac{(1.997 \times 10^3)^2 \ (8.96 \times 10^{-2}) \ (.284)}{(30 \times 10^6) \ (116.2 \times 10^{-6}) \ (386)} \right]^{.25} = .523$$

$kl = 4.73$

Substituting in the frequency equation from Appendix III (i.e. 1 = cos kl cosh kl)

$1 \doteq (.01745)\ (56.65)$

$1 \doteq .99$ (within slide rule accuracy)

Hence shear and rotary inertia show no significant effect on theoretical frequency of the tested beams considering both Kruszewski's and Timoshenko's works.

CHAPTER IV

RESULTS

(1) Results are tabulated in table No. 1. Errors from the theoretical natural frequency ranged from about 9 - 30%.

(2) Shear and rotary inertia had negligible effect on the natural frequencies of the beams tested.

(3) Deviation was almost uniformly greater for longer beams than for shorter beams of the same cross section.

Beam No.	f Theor.	f Obs.	Diff.	% Error
1-1	202	142.5	59.5	29.5
1-2	263	237.5	25.5	10.32
2-1	226	185	41	18.15
2-2	256	210	46	17.96
2-3	283	240	43	15.2
2-4	317	267.5	49.5	15.63
2-5	357	325	32	8.96
3-1	201	140	61	30.4
3-2	219.5	200	19.5	8.88
3-3	243	210	33	13.58
3-4	268	230	38	14.18

TABLE NO. 1

CHAPTER V

CONCLUSIONS

(1) Uniform beams of rectangular cross section, small in thickness and clamped at both ends, with the thickness in the plane of loading, can be expected to vary on the average approximately 14% lower than the theoretically predicted value. Of this, practically none can be attributed to shear and rotary inertia for frequencies under 270 cps.

(2) Excitation can be accomplished electro-magnetically.

(3) It is felt by the author that the end conditions account for the remaining variation from the theoretically predicted value of the natural frequency. The ideally built in beam was not closely enough approached in this experiment - clamping two faces between two large blocks and doweling the beam does not approach closely enough a beam contained on all sides and the end. Also, there is undoubtedly an elastic deformation of the supports in the area where the beam enters the supports, which again moves us away from the ideal, which assumes infinite rigidity of supports.

(4) In trying to reconcile the data with theoretical frequencies, the following possibility was considered:

> the bending moment at the end was proportional to the slope of the beam at the end. This could be the case if the bolts stretched.

This assumption does not seem to be borne out by the data.

BIBLIOGRAPHY

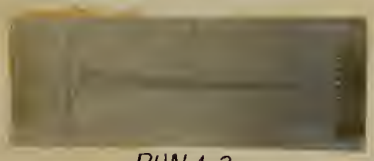
(1) Den Hartog, J. P. MECHANICAL VIBRATIONS, 3rd Edition McGraw-Hill, 1947

(2) Timoshenko, S. VIBRATION PROBLEMS IN ENGINEERING, 2nd Edition, D. van Nostrand, 1937

(3) National Advisory Committee for Aeronautics, Technical Note 1909 of July 1949

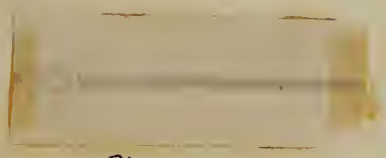
APPENDIX I

OSCILLOGRAPH TRACES AND TABLES 2, 3, AND 4

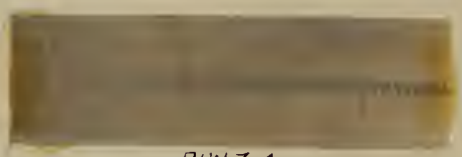


RUN 1-2
$f_{OBS.} = 237.5$ CPS
$f_{THEOR} = 263$ "

RUN 2-1
$f_{OBS} = 185$ CPS
$f_{THEOR} = 226$ "

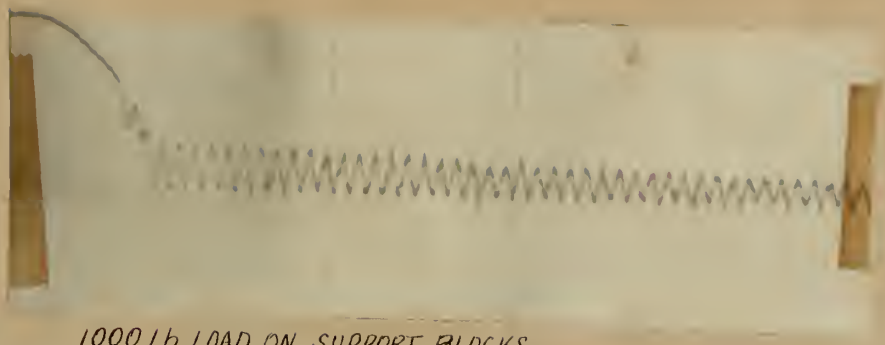


RUN 2-5
$f_{OBS} = 325$ CPS
$f_{THEOR} = 357$ "

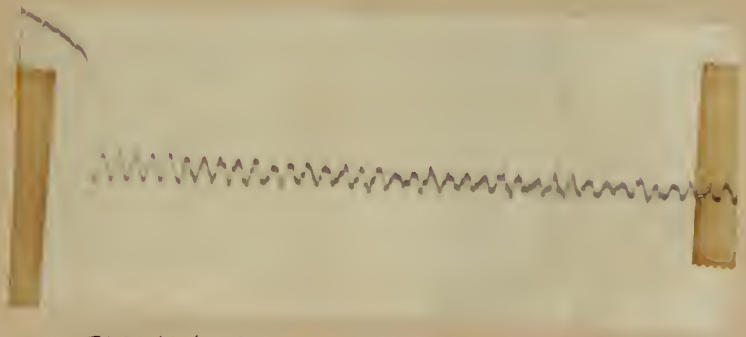


RUN 3-4
$f_{OBS} = 230$ CPS
$f_{THEOR} = 268$ "

FIG. 2



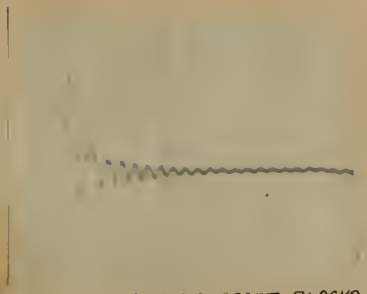
1000 Lb LOAD ON SUPPORT BLOCKS
f_{OBS} = 110 CPS



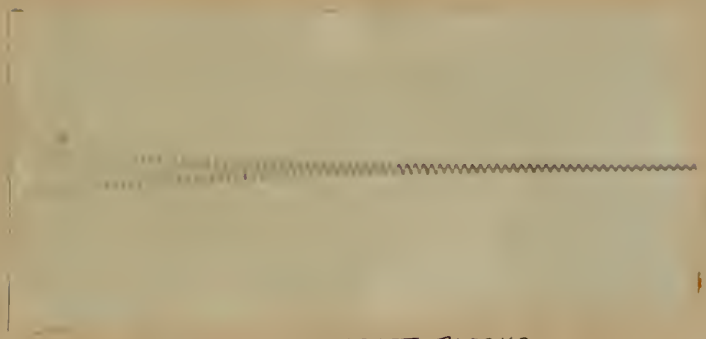
5000 Lb LOAD ON SUPPORT BLOCKS
f_{OBS} = 110 CPS

BEAM NO 3-1

FIG 3



1000 Lb LOAD ON SUPPORT BLOCKS
f_{OBS} = 140 CPS



4000 Lb LOAD ON SUPPORT BLOCKS
f_{OBS} = 220 CPS

BEAM NO 3-4

FIG. 4

1000 Lb. LOAD ON SUPPORT BLOCKS
f_{bs} = 140 CPS

BEAM NO. 4

FIG. 5

BEAM No. 3-1

Load on Supports	f Act.	f Theor.	Diff.	% Error
300 lbs	110	201	91	45.3
1000 lbs	110	201	91	45.3
3000 lbs	110	201	91	45.3
5000 lbs	110	201	91	45.3
10000 lbs	105	201	96	47.8

TABLE No. 2

BEAM No. 3-4

Load on Supports	f Act.	f Theor.	Diff.	% Error
300 lbs	133	268	135	50.3
1000 lbs	140	268	128	47.8
5000 lbs	135	268	133	49.7
10000 lbs	133	268	135	50.3

TABLE No. 3

BEAM No. 4

Load on Supports	f Act.	f Theor.	Diff.	% Error
300 lbs	125	178	53	29.8
1000 lbs	140	178	38	21.4
2000 lbs	210	178	-32	-18.
3000 lbs	220	178	-42	-23.6
4000 lbs	220	178	-42	-23.6
6000 lbs	230	178	-52	-29.2

TABLE No. 4

APPENDIX II

BEAM DIMENSIONS

BEAM NO.	WIDTH	THICKNESS	CROSS SECTION AREA	MOMENT INERTIA
1	.4754"	.124"	$5.89 \times 10^{-2} in^2$	$7.55 \times 10^{-5} in^4$
2	.7178"	.1248"	$8.96 \times 10^{-2} in^2$	$11.62 \times 10^{-5} in^4$
3	.9620"	.1238"	$11.9 \times 10^{-2} in^2$	$15.19 \times 10^{-5} in^4$

SPECIFIC WEIGHT .284 lb per cu. in.

BEAM NO. 1

RUN NO.	LENGTH
1-1	11.31"
1-2	9.92"

BEAM NO. 2

2-1	10.70"
2-2	10.06"
2-3	9.57"
2-4	9.02"
2-5	8.52"

BEAM NO. 3

3-1	11.33"
3-2	10.83"
3-3	10.31"
3-4	9.80"

BEAM NO. 4

Width = 1.0225"
Thickness = .0312"
Length = 6.04"
Cross Section Area = $3.19 \times 10^{-2} in^2$
Area Moment Inertia = $2.59 \times 10^{-6} in^4$

$$\omega_{\text{Theor.}} = \frac{(22.4)}{(6.04)^2} \sqrt{\frac{(30 \times 10^6)\,(2.59 \times 10^{-6})\,(386)}{(.284)\,(3.19 \times 10^{-2})}} = 1118 \text{ rads/sec}$$

$$f_{\text{Theor.}} = \frac{1118}{2\pi} = 178 \text{ cps}$$

APPENDIX III

FREQUENCY EQUATION TAKING INTO ACCOUNT SHEAR AND ROTARY INERTIA

EFFECT OF TRANSVERSE SHEAR AND ROTARY

The effect of shear and rotary inertia were determined by modifying the results of E. T. Kruszewski in his article "Effect of Transverse Shear and Rotary Inertia on the Natural Frequency of a Uniform Beam" published in the NACA Technical Note 1909 of July 1949.

In his article Kruszewski includes the effects of shear and rotary inertia to give an equilibrium equation as follows:

$$EI\frac{d^4y}{dx^4}+EI\left(\frac{mw^2}{A_sG}+\frac{mw^2}{A_TE}\right)\frac{d^2y}{dx^2}-\left(mw^2-\frac{Im^2w^4}{A_TA_SG}\right)y=0$$

He then finds the solution to be

$$y = c_1\cosh K_B\alpha\frac{x}{l}+C_2\sinh K_B\alpha\frac{x}{l}+C_3\cos K_B\beta\frac{x}{l}+C_4\sin K_B\beta\frac{x}{l}$$

His definitions are as follows:

A_s	- effective shear area
A_T	- effective total cross section
m	- mass of beam per unit length
ω	- natural frequency
E	- modulus of elasticity (psi)
I	- moment of inertia (in^4)
G	- shear modulus (psi)
L	- half length of a free-free beam

$$K_B = \omega \sqrt{\frac{mL^4}{EI}} \qquad K_s = \frac{1}{L} \sqrt{\frac{EI}{A_s G}}$$

$$K_{RI} = \frac{1}{L} \sqrt{\frac{I}{A_T}}$$

$$\alpha = \sqrt{\frac{-(K_s^2 + K_{RI}^2) + \sqrt{(K_s^2 - K_{RI}^2)^2 + \frac{4}{K_B^2}}}{2}}$$

$$\beta = \sqrt{\frac{(K_s^2 + K_{RI}^2) + \sqrt{(K_s^2 - K_{RI}^2)^2 + \frac{4}{K_B^2}}}{2}}$$

Kruszewski then applies boundary conditions to his solution to obtain the frequency equation for a cantilever, symmetrically vibrating free-free beam, and an antisymetrically vibrating free-free beam respectively.

It was the author's desire to have a frequency equation for a clamped-clamped beam; this was obtained as follows.

$$(1)\quad y = c_1 \cosh K_B \alpha \frac{x}{L} + C_2 \sinh K_B \alpha \frac{x}{L} + C_3 \cos K_B \beta \frac{x}{L} + C_4 \sin K_B \beta \frac{x}{L}$$

$$(2)\quad \overset{\circ}{y} = C_1 K_B \frac{\alpha}{L} \sinh K_B \alpha \frac{x}{L} + C_2 K_B \frac{\alpha}{L} \cosh \alpha \frac{x}{L} - C_3 K_B \frac{\beta}{L} \sin K_B \beta \frac{x}{L} + C_4 K_B \frac{\beta}{L} \cos K_B \beta \frac{x}{L}$$

$$(3)\quad \overset{\circ\circ}{y} = C_1 \frac{K_B^2 \alpha^2}{L^2} \cosh K_B \alpha \frac{x}{L} + C_2 \frac{K_B^2 \alpha^2}{L^2} \sinh K_B \alpha \frac{x}{L} - C_3 \frac{K_B^2 \beta^2}{L^2} \cos K_B \beta \frac{x}{L} - C_4 \frac{K_B^2 \beta^2}{L^2} \sin K_B \beta \frac{x}{L}$$

$$(4)\quad \overset{\circ\circ\circ}{y} = C_1 \frac{K_B^3 \alpha^3}{L^3} \sinh K_B \alpha \frac{x}{L} + C_2 \frac{K_B^3 \alpha^3}{L^3} \cosh K_B \alpha \frac{x}{L} + C_3 \frac{K_B^3 \beta^3}{L^3} \sin K_B \beta \frac{x}{L} - C_4 \frac{K_B^3 \beta^3}{L^3} \cos K_B \beta \frac{x}{L}$$

Boundary Conditions:

At $x = 0$ $\quad y = 0$

$x = 2L$ $\quad y = 0$

$$\text{At } \begin{matrix} x = 0 \\ x = 2L \end{matrix} \Bigg\} \frac{1}{L}\frac{dy}{dx} = \frac{1}{A_s G}\left[V\right]_{\substack{x=0 \\ x=2L}} = -\frac{1}{L}\frac{K_s^2 K_B^2}{1-K_s^2 K_{RI}^2 K_B^2}\left[\frac{1}{K_B^2}\frac{d^3y}{dx^3} + (K_s^2 + K_{RI}^2)\frac{dy}{dx}\right]_{\begin{matrix} x=0 \\ x=2L \end{matrix}}$$

Applying these boundary conditions:

(A) $0 = C_1 + C_3$

(B) $0 = C_1 \cosh 2K_B\alpha + C_2 \sinh 2K_B\alpha + C_3 \cos 2K_B\beta + C_4 \sin 2K_B\beta$

(C) $$0 = C_2\left\{\left[\frac{K_B\alpha^3}{L^3} + (K_s^2+K_{RI}^2)\frac{K_B\alpha}{L}\right]\left(\frac{K_s^2K_B^2}{1-K_s^2K_{RI}^2K_B^2}\right) + \frac{K_B\alpha}{L}\right\}$$
$$+ C_4\left\{\left[(K_s^2 + K_{RI}^2)\frac{K_B\beta}{L} - \frac{K_B\beta^3}{L^3}\right]\left(\frac{K_s^2K_B^2}{1-K_s^2K_{RI}^2K_B^2}\right) + \frac{K_B\beta}{L}\right\}$$

(D) $$0 = (C_1 \sinh 2K_B\alpha)\left[\frac{K_B\alpha}{L} + \left(\frac{K_s^2K_B^2}{1-K_s^2K_{RI}^2K_B^2}\right)\left(\frac{K_B\alpha^3}{L^3} + (K_s^2+K_{RI}^2)\frac{K_B\alpha}{L}\right)\right]$$
$$+ (C_2 \cosh 2K_B\alpha)\left[\frac{K_b\alpha}{L} + \left(\frac{K_B\alpha^3}{L^3} + (K_s^2+K_{RI}^2)\frac{K_B\alpha}{L}\right)\left(\frac{K_s^2K_B^2}{1-K_s^2K_{RI}^2K_B^2}\right)\right]$$
$$+ (C_3 \sin 2K_B\beta)\left[\frac{-K_B\beta}{L} + \left(\frac{K_s^2K_B^2}{1-K_s^2K_{RI}^2K_B^2}\right)\left(\frac{K_B\beta^3}{L^3} - (K_s^2+K_{RI}^2)\frac{K_B\beta}{L}\right)\right]$$
$$+ (C_4 \cos 2K_B\beta)\left[\frac{K_B\beta}{L} - \left(\frac{K_s^2K_B^2}{1-K_s^2K_{RI}^2K_B^2}\right)\left(\frac{K_B\beta^3}{L^3} - (K_s^2+K_{RI}^2)\frac{K_B\beta}{L}\right)\right]$$

The characteristic values are obtained by setting the determinant of the coefficients of the "C's" equal to zero.

$$0 = \begin{vmatrix} 1 & 0 & 1 & 0 \\ \cosh 2K_B & \sinh 2K_B & \cos 2K_B & \sin 2K_B \\ 0 & (1) & 0 & (2) \\ (3) & (4) & (5) & (6) \end{vmatrix}$$

$$(1)\ \left\{\left[\frac{K_B\alpha^3}{L^3} + (K_s^2 + K_{RI}^2)\ \frac{K_B\alpha}{L}\right]\left(\frac{K_s^2K_B^2}{1-K_s^2K_{RI}^2K_B^2}\right) + \frac{K_B\alpha}{L}\right\}$$

$$(2)\ \left\{\left[(K_s^2 + K_{RI}^2)\ \frac{K_B\beta}{L} - \frac{K_B\beta^3}{L}\right]\left(\frac{K_s^2K_B^2}{1-K_s^2K_{RI}^2K_B^2}\right) + \frac{K_B\beta}{L}\right\}$$

$$(3)\ \left[\frac{K_B\alpha}{L} + \frac{K_s^2K_B^2}{1-K_s^2K_{RI}^2K_B^2}\left(\frac{K_B\alpha^3}{L^3} + (K_s^2 + K_{RI}^2)\ \frac{K_B\alpha}{L}\right)\right] \sinh 2K_B\alpha$$

$$(4)\ \left[\frac{K_B\alpha}{L} + \left(\frac{K_B\alpha^3}{L^3} + (K_s^2 + K_{RI}^2)\ \frac{K_B\alpha}{L}\right)\frac{K_s^2K_B^2}{1-K_s^2K_{RI}^2K_B^2}\right] \cosh 2K_B\alpha$$

$$(5)\ \left[\frac{-K_B\beta}{L} + \frac{K_s^2K_B^2}{1-K_s^2K_{RI}^2K_B^2}\left(\frac{K_B\beta^3}{L^3} - (K_s^2 + K_{RI}^2)\ \frac{K_B\beta}{L}\right)\right] \sin 2K_B\beta$$

$$(6)\ \left[\frac{K_B\beta}{L} - \frac{K_s^2K_B^2}{1-K_s^2K_{RI}^2K_B^2}\left(\frac{K_B\beta^3}{L^3} - (K_s^2 + K_{RI}^2)\ \frac{K_B\beta}{L}\right)\right] \cos 2K_B\beta$$

Trial and error was employed by the author to solve for the in this fourth order determinant.

It might be noted that Kruszewski in his article sets his slope per unit length equal to the shearing strain per unit length as a boundary condition at $x = 0$ and $x = 2L$. Timoshenko prefers to use slope equal to zero at both ends. Evaluation considering Timoshenko's derivation and end conditions was therefore considered advisable.

On pages 337-338 of Timoshenko's "Vibration Problems in Engineering", he derives the following differential equation for the lateral vibration of prismatical bars:

$$EI\frac{\partial^4 y}{\partial x^4}+\frac{\gamma A}{g}\frac{\partial^2 y}{\partial t^2}-\left(\frac{\gamma I}{g}+\frac{EI\gamma}{gk'G}\right)\frac{\partial^4 y}{\partial x^2 \partial t^2}+\frac{\gamma I}{g}\frac{\gamma}{gk'G}\frac{\partial^4 y}{\partial t^4}=0$$

if a solution $y = XT$ is assumed where

$$X = C_1 \sin kx + C_2 \cos kx + C_3 \sinh kx + C_4 \cosh kx$$

$$T = (A \cos pt + B \sin pt)$$

And if we keep in mind that Kruszewski's

$\omega = p$ in Timoshenko

$m = \frac{A}{g}$, the above differential equation transforms to

$$EI\frac{\partial^4 y}{\partial x^4}+EI\left(\frac{m\omega^2}{Ak'G}+\frac{m\omega^2}{EA}\right)\frac{\partial^2 y}{\partial x^2}-\left(m\omega^2-\frac{m^2 I\omega^4}{A^2 k' G}\right)y=0$$

which is the same as Kruszewski has.

The symbols in Timoshenko's equation are defined as follows:

E	=	Young's Modulus of elasticity
I	=	moment of inertia of cross section of beam
g	=	gravitational constant
G	=	shear modulus
γ	=	weight of material of bar per unit volume
k^1	=	numerical factor depending on shape of cross section
k	=	$(\frac{p^2 A\gamma}{EIg})^{\frac{1}{4}}$
A	=	area of cross section of beam
l	=	length of beam

$$\frac{dX}{dx} = C_1 (-k \sin kx + k \sinh kx) + C_2 (-k \sin kx - k \sinh kx)$$
$$+ C_3 (k \cos kx + k \cosh kx) + C_4 (k \cos kx - k \cosh kx)$$

Boundary Conditions:

(1) $(X)_{x=o} = 0$ (2) $\left(\frac{dX}{dx}\right)_{x=o} = 0$

(3) $(X)_{x=l} = 0$ (4) $\left(\frac{dX}{dx}\right)_{x=l} = 0$

from (1) $C_1 = 0$

(2) $C_3 = 0$

from (1), (2), & (3); $C_2 (\cos kl - \cosh kl) + C_4 (\sin kl - \sinh kl) = 0$

from (1), (2), & (4); $C_2 (-k \sin kl - k \sinh kl) + C_4 (k \cos kl - k \cosh kl) = 0$

A solution for C_2 and C_4 other than zero can be had only if

$$\begin{vmatrix} (\cos kl - \cosh kl) & (\sin kl - \sinh kl) \\ (-k \sin kl - k \sinh kl) & (k \cos kl - k \cosh kl) \end{vmatrix} = 0$$

Evaluation of this determinant leads to the frequency equation

$$1 = \cos kl \cosh kl$$

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